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A MILITARY TRUCK-TRANSPORT MODEL THEORETICAL DERIVATION OF GENERAL TIME DEPENDENT TRANSFER EQUATIONS AND DENSITY FUNCTIONS

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RESEARCH REPORT

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A MILITARY TRUCK-TRANSPORT MODEL

THEORETICAL DERIVATION
OF
GENERAL TIME-DEPENDENT TRANSFER EQUATIONS
AND
DENSITY FUNCTIONS

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by

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PREFACE

This is the first report in a series of studies conducted to produce a realistic operations research theory of attrition and reliability of land supply carriers. This research is a part of the Generalized Transportation Analogue (GTA) effort. The purpose of the "Truck-Transport Model" study is to test conclusively the viability of (and, if viable, to exploit) the hypothesis that general terrain properties, enemy action, personnel behavior, and limited command decision determine the most probable performance of a land supply system and that this performance can be adequately described with transfer theory.

Chapter 1 states the basic concepts of transfer theory, develops the subsidiary terms, and derives the formulation relating to these terms. Not explicitly stated is the assumption that the processes which such a model describes are not strictly "mechanistic", but that the "on-going events" are affected by the ensemble of past occurrences.

Chapter 2 explains the subsidiary terms employed in Chapter 1 and relates them to the systems research problem under study. Equations 2.6.3 express the final form of the theory and are amenable to the following interpretation:

> They determine the number of loaded trucks in transit per unit area at a given place at a given time which, in the course of carrying out a mission, are being subjected to being put out of operation, deflected, or forced to change their speed by terrain, enemy action, or decision, barring historical accident.

In the succeeding chapters of the report, this theory will be tested and applied against theoretical, practical, and historical evidence.

CHAPTER 1 - THE TRANSFER EQUATIONS

INTRODUCTION

Today, military mobility is becoming identified with a major problem area: logistics of motor transport. A further simplifying assumption frequently made is that this problem can be measured by measuring the ability of a vehicle to traverse natural terrain. This paper contends that the validity of this implicit assumption has not been demonstrated; rather, the complete system should be analyzed, with simplifying assumptions made only if the model complexity is beyond the capacity of modern mathematics and computer technology.

This paper will present the derivation of the theoretical model. The importance of the parameters will be the subject of a computer analysis after the model has been historically verified. The companion paper "The Terrain Scattering Probability" is prepared separately so that the requirement for a measure of performance for specific units may be met but will enter this model as the scattering parameter.

GENERAL NATURE OF THE PROBLEM*

It is desired to know the transfer intensity and the time-space density of units of supply under large-scale operations involving varying battle conditions and terrain features. We assume that general coefficient and probability functions that take these factors into account can be realistically defined. Here, we will investigate the intensity and density of loaded carrier units. In general, such an ensemble can not be adequately described in ordinary space, but requires a six-dimensional place space μ $(\bar{x}_1, x_2, x_3, v_{x1}, v_{x2}, v_{x3})$. This problem requires only a four-dimensional place space, which will be modified to suit the problem.

DEFINITIONS

Consider a place space μ with coordinates $\mu(\vec{R}, \vec{\Omega}, t)$.

Variables: Position $= \vec{R}$

Direction $= \vec{\Omega} \quad (\| \vec{\Omega} \| = 1)$.

Dependent Variable: I_v = intensity.

*This development parallels Chandrasekhar, Reference 3, and follows Anselone, Reference 6. (References are given in the Bibliography.)

1.2.1 $I_v(\vec{R}, \vec{\Omega})$ = units at $\vec{R}$ traveling in the direction $\vec{\Omega}$

per unit area $\perp$ to $\vec{\Omega}$
per unit solid angle about $\vec{\Omega}$
per second

where the units are loaded trucks.

For a given $\vec{R}$ and $\vec{\Omega}$, construct an element of area $d\sigma \perp$ to $\vec{\Omega}$. With each point of $d\sigma$ as a vertex, construct a solid angle $d\omega$ about $\vec{\Omega}$. The resultant figure is called a pencil.

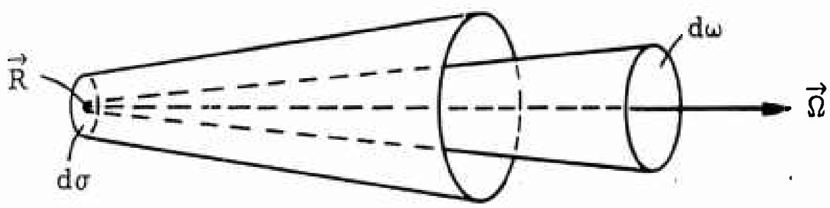
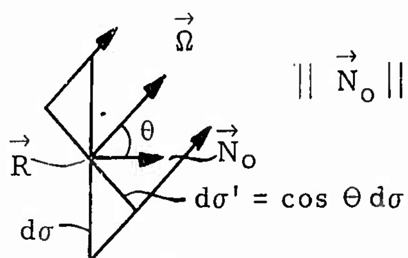


Figure 1. Pencil.

From the definition of $I_v(\vec{R}, \vec{\Omega})$, we obtain

1.2.2 $I_v(\vec{R}, \vec{\Omega})\, d\sigma\, d\omega\, dt$ = units at $\vec{R}$ traveling through points of $d\sigma$ with directions in $d\omega$ about $\vec{\Omega}$.

Let $d\sigma$ be an area not necessarily $\perp$ to $\vec{\Omega}$.



$\|\vec{N}_o\| = 1$

Figure 2.

The radiation passing through $d\sigma$ also passes through $d\sigma'$. Thus,

1.2.3 $I_v(\vec{R}, \vec{\Omega}) \cos \Theta\, d\sigma\, d\omega\, dt$ = units at $\vec{R}$ traveling through points of $d\sigma$ with direction in $d\omega$ about $\vec{\Omega}$.

It is convenient to define

1.2.4 $\vec{I}_v(\vec{R}, \vec{\Omega}) = I_v(\vec{R}, \vec{\Omega})\, \vec{\Omega}$, which is a vector at $\vec{R}$ in the $\vec{\Omega}$ direction having a length equal to the intensity in that direction.

The net vector flux $\pi \vec{F}_V (\vec{R})$ is defined

1.2.5 $\quad \pi \vec{F}_V (\vec{R}) = \int_\omega \vec{I}_V \cdot \vec{n}_o \vec{n}_o \, d\omega = \int_\omega I_V \vec{\Omega} \cdot \vec{n}_o \vec{n}_o \, d\omega.$

The net transfer of units across a unit area in any arbitrary direction $\vec{N}$ at $\vec{R}$ ($||N|| = 1$) is given by

1.2.6 $\quad \pi \vec{F}_V \cdot N = \int_\omega \vec{I}_V \cdot \vec{n}_o \vec{n}_o \cdot \vec{N} \, d\omega = \int_\omega I \vec{\Omega} \cdot \vec{n}_o \vec{n}_o \cdot \vec{N} \, d\omega$

and $\quad || \pi \vec{F}_V || = \int_\omega I_V \cos \Theta \, d\omega$.

where Θ is the angle between the chosen direction $\vec{N}$ and the variable (of integration) direction.

Consider the units associated with $I_V (\vec{R}, \vec{\Omega})$ for a particular $\vec{R}$ and $\vec{\Omega}$. As the units proceed in the $\vec{\Omega}$ direction, they may be absorbed or scattered in a direction or velocity other than the direction and velocity of interest.

Thus, the intensity is diminished as it travels through the medium.

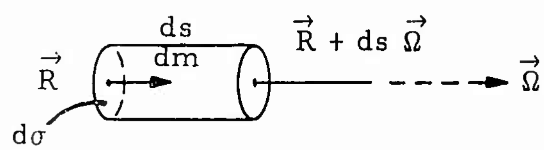


Figure 3.

Let $I_V (\vec{R} + s\vec{\Omega}, \vec{\Omega})$ denote the number of units of $I_V (\vec{R}, \vec{\Omega})$ that remain in the ray after traveling a distance s in $\vec{\Omega}$ direction.

The difference

$$d I_V = I_V (\vec{R} + ds\,\vec{\Omega}, \vec{\Omega}) - I_V (\vec{R}, \vec{\Omega}) \leq 0$$

is generally proportional to ds, to I_V, and to the density ρ of the medium. Thus, write

1.2.7 $\quad d I_V = - k_V \rho I_V \, ds$

where k_V is the "Mass Absorption Coefficient".

Consider the loss from $I_V (\vec{R}, \vec{\Omega})$ due to absorption in an element of "mass",

$$dM = \rho \, d\sigma \, ds \text{ (see Figure 3).}$$

Recalling the definition of $I_v(\vec{R}, \vec{\Omega})$ and equation 1.2.2., the losses of units in dM are given by

1.2.8 $I_v\, d\sigma \cdot k_v \rho\, ds\, d\omega\, dt = k_v\, I_v\, dM\, d\omega\, dt$ = loss of units of velocity v, in direction $\vec{\Omega}$ in dM

Suppose that a unit traveling in the $\vec{\Omega}$ direction is absorbed at $\vec{R}$. We define the Phase Function, $p(\vec{\Omega}, \vec{\Omega}')$, such that

1.2.9 $\frac{p(\vec{\Omega}, \vec{\Omega}')}{4\pi}$ = the probability per unit solid angle about $\vec{\Omega}'$ that the unit will be scattered into the $\vec{\Omega}$ direction.

Thus,

1.2.10 $\frac{p(\vec{\Omega}, \vec{\Omega}')}{4\pi}\, k_v\, I_v\, (\vec{R}, \vec{\Omega}')\, dt\, dM\, d\omega\, d\omega'$
= units scattered from direction $\vec{\Omega}'$ into direction $\vec{\Omega}$.

Integrating over all directions $\vec{\Omega}'$ of the incident ray,

1.2.11 $j_v^{(s)} = \frac{k_v}{4\pi} \int_{\omega'} p\, (\vec{\Omega}, \vec{\Omega}')\, I_v\, (\vec{R}, \vec{\Omega}')\, d\omega'$

= units scattered from all directions $\vec{\Omega}'$ into the particular direction $\vec{\Omega}$

per unit solid angle about $\vec{\Omega}$
per second
per unit mass at $\vec{R}$.

$j_v^{(s)}$ is the emission coefficient due to scattering. The general emission coefficient is

1.2.12 $j_v = j_v\, (\vec{R}, \vec{\Omega})$ = units emitted in direction $\vec{\Omega}$
per unit solid angle about $\vec{\Omega}$
per second
per unit mass at $\vec{R}$.

The ratio

1.2.13 $j\, k_v = \mathcal{J}_v = \mathcal{J}_v\, (\vec{R}, \vec{\Omega})$ is the source function.

DERIVATION OF THE ELEMENTARY EQUATION OF TRANSFER

Consider an element of mass dM emitting in the $\vec{\Omega}$ direction.

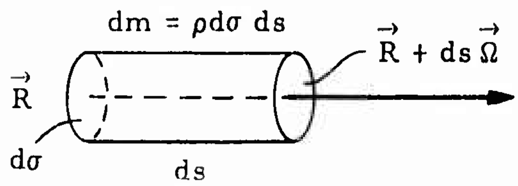


Figure 4.

From the definition of emission coefficient, we have

1.3.1 $j_\nu\, dM = j_\nu\, \rho\, d\sigma\, ds$ = units emitted by dM in the $\vec{\Omega}$ direction

per unit solid angle about $\vec{\Omega}$, per second.

Dividing by $d\sigma$ and letting $d\sigma \to 0$, we obtain

1.3.2 $j_\nu \rho\, ds$ = units emitted along the line $(\vec{R},\ \vec{R} + ds\,\vec{\Omega})$ in the $\vec{\Omega}$ direction

per unit solid angle about $\vec{\Omega}$
per unit area $\perp \vec{\Omega}$
per second.

By definition,

$j_\nu \rho\, ds$ = intensity of direction $\vec{\Omega}$ emitted along the segment $(\vec{R},\ \vec{R} + ds\,\vec{\Omega})$.

Taking into account absorption and emission, the net change in the intensity is

1.3.3 $d\, I_\nu\, (\vec{R},\ \vec{\Omega})$.

Thus, the transfer equation is

1.3.4 $$\frac{1}{k_\nu \rho}\ \frac{d I_\nu (\vec{R},\ \vec{\Omega})}{ds} = I_\nu\ (\vec{R},\ \vec{\Omega}) - \mathcal{J}_\nu\ (\vec{R},\ \vec{\Omega})$$

or

1.3.5 $$\frac{1}{k_\nu\, \rho}\ \vec{\Omega} \cdot \nabla I_\nu\ (\vec{R}, \vec{\Omega}) = I_\nu\ (\vec{R}, \vec{\Omega}) - \mathcal{J}_\nu\ (\vec{R}, \vec{\Omega}).$$

Remember that $\frac{d}{ds}$ and ∇ are operators for the variable $\vec{R}$ and arbitrary but fixed $\vec{\Omega}$.

THE DENSITY OF UNITS

The density U_v of units of the radiation at velocity v at any given point is the number of units per unit volume that are in transit in the immediate neighborhood of the point considered.

Following Chandrasekhar (Reference 3, Chapter 1, Section 2.3) to find this density at a point P, construct around P an infinitesimal volume V with a convex surface σ. Around V construct another convex surface Σ such that the linear dimensions of Σ are large compared with those of σ and such that the volume inclosed by Σ is nevertheless small enough such that the intensity in any given direction can be considered as the same for all points inside Σ.

Now the radiation traversing the volume V must have crossed some element of the surface Σ. Let $d\Sigma$ be such an element. Further let Θ and ζ denote the angles which the normal to $d\Sigma$ and to an element $d\sigma$ of σ makes with the lines joining the two elements. From equation 1.2.3, the units streaming across $d\Sigma$ that also flow across $d\sigma$ are

$$1.4.1 \quad I_v \cos\Theta \, d\Sigma \, d\omega' = I_v \frac{\cos\zeta \cos\Theta \, d\sigma \, d\Sigma}{r^2}$$

since the solid angle $d\omega'$ subtended by $d\sigma$ at $d\Sigma$ is $\frac{d\sigma \cos\zeta}{r^2}$ where r is the distance between $d\Sigma$ and $d\sigma$.

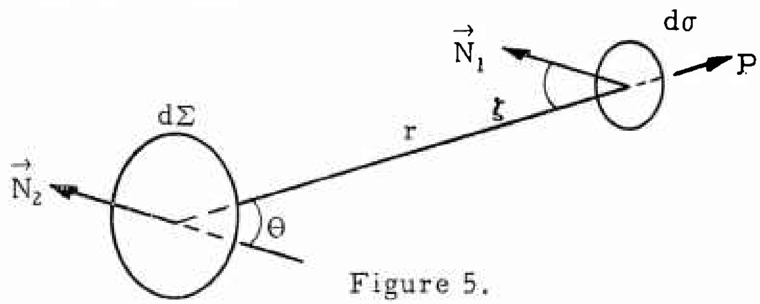


Figure 5.

Let ℓ be the length traversed by the pencil of radiation through the volume element V. The radiation 1.4.1 incident on $d\sigma$ per second will traverse the element in a time $\frac{\ell}{v}$, where v is the speed of the units. The contribution to the total amount of radiating units in course of transit through v by the pencil is

$$1.4.2 \quad I_v \frac{\cos\zeta \cos\Theta \, d\sigma \, d\Sigma}{r^2} \frac{\ell}{v} = \frac{1}{v} I_v \, dv \, d\omega$$

where $d\omega = d\Sigma \frac{\cos\Theta}{r^2}$ is the solid angle subtended by $d\Sigma$ at P and $dV = d\sigma \cos\zeta$ is the volume intercepted in V by the pencil of radiation. Thus, the total number of units in course of transit through x due to streaming in from all directions can be obtained by integrating

1.4.2 over all V and ω. Hence,

1.4.3 $\frac{1}{v} \int dV \int d\omega \, I_v = \frac{V}{v} \int I_v \, d\omega$.

Thus,

1.4.4 $U_v = \frac{1}{v} \int I_v \, d\omega$.

Let $J_v = \frac{1}{4\pi} \int I_v \, d\omega$ be the average intensity. Then

1.4.5 $U_v = \frac{4\pi}{v} J_v$.

NOTES ON THE SPECIFIC TRANSPORT EQUATION

Following the spirit of the simple transport model discussed, the specific transport model in which we are interested has the following terms:

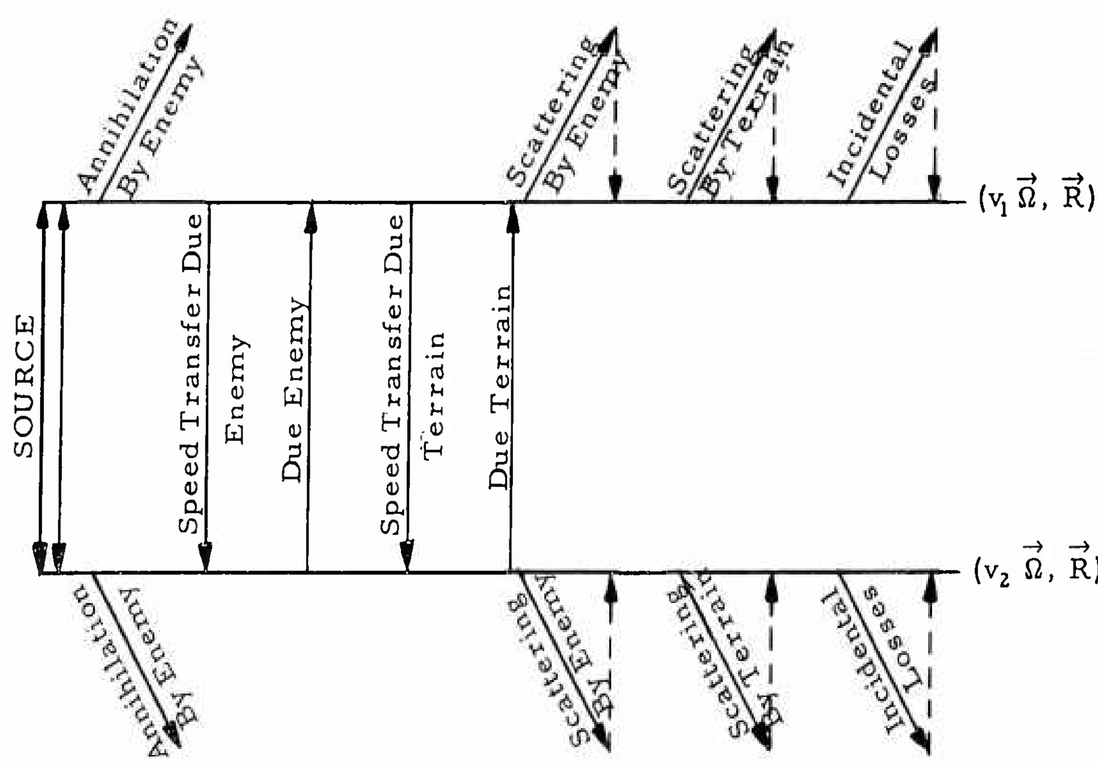


Figure 6. Model.

EVENT AND INTERACTION CROSS SECTIONS

(See Figure 6)

<u>Absorption (losses)</u>		
Annihilation by enemy action	$k_{av1} \rho_{av1}$	$k_{av2} \rho_{av2}$
Speed transfer (enemy action)	$k_{bv1} \rho_{av1}$	$k_{bv2} \rho_{av2}$
Speed transfer (terrain)	$k_{cv1} \rho_{bv1}$	$k_{cv2} \rho_{bv2}$
Scattering (enemy action)	$k_{dv1} \rho_{av1}$	$k_{dv2} \rho_{av2}$
Scattering (terrain)	$k_{ev1} \rho_{bv1}$	$k_{ev2} \rho_{ev2}$
Incidental losses	L_1	L_2
<u>Gains</u>		
Sources	S_1	S_2
Speed transfer (enemy action)	$k_{bv2} \rho_{av2} \; P_a (v_2 \rightarrow v_1, \vec{\Omega}, \vec{\Omega}')$	$k_{bv1} \rho_{av1} \; P_a (v_1 \rightarrow v_2, \vec{\Omega}, \vec{\Omega}')$
Speed Transfer (terrain)	$k_{cv2} \rho_{bv2} \; P_b (v_2 \rightarrow v_1, \vec{\Omega}, \vec{\Omega}')$	$k_{cv1} \rho_{bv1} \; P_b (v_1 \rightarrow v_2, \vec{\Omega}, \vec{\Omega}')$
Scattering (enemy action)	$k_{dv1} \rho_{av1} \; P_{cv1} (\vec{\Omega}, \vec{\Omega}')$	$k_{dv2} \rho_{av2} \; P_{cv2} (\vec{\Omega}, \vec{\Omega}')$
Scattering (terrain)	$k_{ev1} \rho_{bv1} \; P_{dv1} (\vec{\Omega}, \vec{\Omega}')$	$k_{ev2} \rho_{ev2} \; P_{dv2} (\vec{\Omega}, \vec{\Omega}')$
Incidental gains	G_1	G_2

where the coefficients and probabilities are defined in the obvious manner with appropriate dimensions.

(A) Absorption:

$$-\left(\left\{k_{av1}+k_{bv1}+k_{dv1}\right\}\rho_{av1}+\left\{k_{cv1}+k_{ev1}\right\}\rho_{bv1}\right)I_{v1}\,ds$$

$$=-K_{v1}\,I_{v1}\,ds$$

and similarly define

$$-K_{v2}\,I_{v2}\,ds=-\left(\left\{k_{av2}+k_{bv2}+k_{dv2}\right\}\rho_{av2}+k_{cv2}\,\rho_{bv2}+k_{ev2}\,\rho_{ev2}\right)I_{v2}\,ds.$$

(B) Emission.

$$j_{\alpha v1}\ (v_2\to v_1)=\frac{K_{bv2}}{4\pi}\int_{\omega'}p_a\,(v_2\to v_1,\vec{\Omega},\vec{\Omega}')\,I_{v2}\,(\vec{R},\vec{\Omega}')\,d\omega',$$

$$j_{\beta v1}\ (v_2\to v_1)=\frac{K_{cv2}}{4\pi}\int_{\omega'}p_b\,(v_2\to v_1,\vec{\Omega},\vec{\Omega}')\,I_{v2}\,(\vec{R},\vec{\Omega}')\,d\omega',$$

$$j_{\gamma v1}=\frac{K_{dv1}}{4\pi}\int_{\omega'}p_{cv1}\,(\vec{\Omega},\vec{\Omega}')\,I_{v1}\,(\vec{R},\vec{\Omega}')\,d\omega',\text{ and}$$

$$j_{\delta v1}=\frac{K_{ev1}}{4\pi}\int_{\omega'}p_{dv1}\,(\vec{\Omega},\vec{\Omega}')\,I_{v1}\,(\vec{R},\vec{\Omega}')\,d\omega'.$$

Thus,

1.5.1 $$dI_{v1}\,(\vec{R},\vec{\Omega})=-K_{v1}\,I_{v1}\,ds+\left(j_{\alpha v1}\left\{v_2\to v_1\right\}\rho_{av2}+j_{\beta v1}\left\{v_2\to v_1\right\}\rho_{bv2}+j_{\gamma v1}\,\rho_{av1}+j_{\delta v1}\,\underline{\underline{\rho_{bv1}}}\right)ds\quad.*$$

Combining similar terms,

1.5.2 $$dI_{v1}\,(\vec{R},\vec{\Omega})=-K_{v1}\,I_{v1}\,ds+j_{v1}\,(v_2\to v_1)\,ds+j_{v1}\,ds$$

where

$$j_{v1}\,(v_2\to v_1)=\left(j_{\alpha v1}\left\{v_2\to v_1\right\}\rho_{av2}+j_{\beta v1}\left\{v_2\to v_1\right\}\rho_{bv2}\right)$$

and

$$j_{v1}=\left(j_{\gamma v1}\,\rho_{av1}+j_{\delta v1}\,\underline{\underline{\rho_{bv1}}}\right).$$

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*In the case of v_2, terrain scattering density ρ_{bv1} becomes ρ_{cv2}.

Finally

$$1.5.3 \quad \boxed{\frac{dI_{v1}\ (\vec{R},\vec{\Omega})}{ds} = -K_{v1}\ I_{v1}\ (\vec{R},\vec{\Omega}) + j_{v1}\ (v_2 \rightarrow v_1) + j_{v1}}.$$

For convenience, write:

$$j_{v1} = \frac{1}{4\pi} \int_{\omega'} \left\{ k_{dv1}\ \rho_{av1}\ p_{cv1}\ (\vec{\Omega}, \vec{\Omega}') + k_{ev1}\ \underline{\underline{\rho_{bv1}}}\ p_{dv1}\ (\vec{\Omega}, \vec{\Omega}') \right\} I_{v1}\ (\vec{R}, \vec{\Omega}')\ d\omega'$$

$$j_{v1}\ (v_2 \rightarrow v_1) = \frac{1}{4\pi} \int_{\omega'} \left\{ k_{bv2}\ \rho_{av2}\ p_a\ (v_2 \rightarrow v_1,\ \vec{\Omega}, \vec{\Omega}') + k_{cv2}\ \rho_{bv2}\ p_b\ (v_2 \rightarrow v_1,\ \vec{\Omega}, \vec{\Omega}') \right\} I_{v2}\ (\vec{R}, \vec{\Omega}')\ d\omega'.$$

Or

$$j_{v1} = \frac{1}{4\pi} \int_{\omega'} p_{v1}\ (\vec{\Omega}, \vec{\Omega}')\ I_{v1}\ (\vec{R}, \vec{\Omega}')\ d\omega' \quad \text{and}$$

$$j_{v1}\ (v_2 \rightarrow v_1) = \frac{1}{4\pi} \int_{\omega'} p_{v1}\ (v_2 \rightarrow v_1,\ \vec{\Omega}, \vec{\Omega}')\ I_{v2}\ (\vec{R}, \vec{\Omega}')\ d\omega'.$$

The appropriate systems of equations now take the form

$$1.5.4 \quad \frac{dI_{v1}\ (\vec{R},\vec{\Omega})}{ds} + K_{v1}\ I_{v1}\ (\vec{R},\vec{\Omega}) = \frac{1}{4\pi} \int_{\omega'} p_{v1}\ (\vec{\Omega},\vec{\Omega}')\ I_{v1}\ (\vec{R},\vec{\Omega}')\ d\omega' + \frac{1}{4\pi} \int_{\omega'} p_{v1}\ (v_2 \rightarrow v_1,\ \vec{\Omega},\vec{\Omega}')\ I_{v2}\ (\vec{R},\vec{\Omega}')\ d\omega'$$

$$1.5.5 \quad \frac{dI_{v2}\ (\vec{R},\vec{\Omega})}{ds} + K_{v2}\ I_{v2}\ (\vec{R},\vec{\Omega}) = \frac{1}{4\pi} \int_{\omega'} p_{v2}\ (\vec{\Omega},\vec{\Omega}')\ I_{v2}\ (\vec{R},\vec{\Omega}')\ d\omega' + \frac{1}{4\pi} \int_{\omega'} p_{v2}\ (v_1 \rightarrow v_2,\ \vec{\Omega},\vec{\Omega}')\ I_{v1}\ (\vec{R},\vec{\Omega}')\ d\omega'.$$

These equations are of the operational form.

$$1.5.6 \quad D\,I_{v1} = k_{v1}\ I_{v1} + A_{v1}\ I_{v2}$$

$$D\,I_{v2} = k_{v2}\ I_{v2} + A_{v2}\ I_{v1}.$$

Or

$$\boxed{\begin{aligned} Dx &= k_{v1}\ x + A_{v1}\ y \\ Dy &= k_{v2}\ y + A_{v2}\ x \end{aligned}}\ .$$

So that

1.5.7
$$\boxed{\begin{aligned} A_{V1}\ y &= Dx - k_{V1}\ x \\ A_{V2}\ x &= Dy - k_{V2}\ y \end{aligned}}\ .$$

This is the general model without arbitrary source or loss terms.

THE TIME-DEPENDENT TRANSFER EQUATIONS

(A) In the simple transfer model, if $I_V\ (\vec{R},\vec{\Omega},\ t)$ is time-dependent, the transfer equation takes the form

1.6.1
$$\frac{1}{k_V\ \rho_V}\ \frac{d\,I\,(\vec{R},\vec{\Omega},t)}{dt} = -\frac{1}{k_V\rho}\ \frac{d\,I_V\,(\vec{R},\vec{\Omega},t)}{ds} - I_V(\vec{R},\vec{\Omega},t) + \tilde{\mathcal{J}}_V(\vec{R},\vec{\Omega},t)\,.$$

(B) In the two-speed transfer problem, if $I_V\,(\vec{R},\vec{\Omega},t)$ is time-dependent, the transfer equations take the form

1.6.2
$$\begin{aligned} \frac{1}{v_1}\ \frac{d\,I_{V1}\ (\vec{R},\vec{\Omega},t)}{dt} = &-\frac{d\,I_{V1}\ (\vec{R},\vec{\Omega},t)}{ds} - K_{V1}\ I_{V1}\ (\vec{R},\vec{\Omega},t) \\ &+\frac{1}{4\pi}\int_{\omega'} p_{V1}\ (\vec{\Omega},\vec{\Omega}')\ I_{V1}\ (\vec{R},\vec{\Omega}',\ t)\ d\omega' \\ &+\frac{1}{4\pi}\int_{\omega'} p_{V1}\ (v_2 \rightarrow v_1\ ,\ \vec{\Omega},\vec{\Omega}')\ I_{V2}\ (\vec{R},\vec{\Omega}',t)\ d\omega' \end{aligned}$$

1.6.3
$$\begin{aligned} \frac{1}{v_2}\ \frac{d\,I_{V2}\ (\vec{R},\vec{\Omega},t)}{dt} = &-\frac{d\,I_{V2}\ (\vec{R},\vec{\Omega},t)}{ds} - K_{V1}\ I_{V1}\ (\vec{R},\vec{\Omega},t) \\ &+\frac{1}{4\pi}\int_{\omega'} p_{V2}\ (\vec{\Omega},\vec{\Omega}')\ I_{V2}\ (\vec{R},\vec{\Omega}',\ t)\ d\omega' \\ &+\frac{1}{4\pi}\int_{\omega'} p_{V2}\ (v_1 \rightarrow v_2\ ,\vec{\Omega},\vec{\Omega}')\ I_{V1}\ (\vec{R},\vec{\Omega}',\ t)\ d\omega', \end{aligned}$$

where arbitrary source and loss terms are deleted.

CHAPTER 2

OPERATIONAL SIGNIFICANCE OF THE TRANSFER MODEL

INTRODUCTION

Before proceeding either to a theoretical investigation of the system of equations 1.5.4 and 1.5.5 or 1.6.2 and 1.6.3 derived in the last chapter or to studying them in a specific geometry, we will relate these systems of integrodifferential equations to the systems research problem at hand through a statement of the significance and dimensions of the coefficients and probabilities involved.

Consider a large-scale theater of operation involving regions of varying terrain conditions and various intensities of enemy action. Suppose that it is desired to be able to predict or maintain a supply system (function) in this area. To do this it is assumed, barring local anomalies and historical accident, that by and large the gross determination of a supply function is governed by general terrain properties, enemy action, personnel behavior, and command decision. In this paper, we will consider only supplies delivered by land carriers such as trucks and will define our unit of supply as one loaded truck. We assume that this unit travels at a speed v and is destination oriented. In the course of a "trip", it may be put out of operation (annihilated), deflected (scattered), or forced to change its speed (transferred) by terrain, enemy action, or decision. Taking such effects into account, the problem is to construct an applicable model that determines a probable supply function.

We have accepted the fact that a natural dependent variable or concept is the intensity (defined in Equation 1.2.1) of units at a point in space traveling in a given direction. A realistic formulation of absorption and scattering requires this angular dependent intensity and requires an integral formulation as, for instance, the intensity in a given direction at a point is influenced by units being scattered into this direction from all other directions at this point. We are thus led in a plausible manner to a transfer equation, and, because of its generality, adopt transfer theory, which we will attempt to make applicable.

In the specific model considered, it is further assumed that cross-country mobility, v_1, substantially differs from on-road mobility, v_2; for this reason, two speed groups are considered. This leads to a system of two dependent transfer equations, one governing the intensity for on-road speed and the other governing the intensity for off-road speed where the average speeds are taken to be representative of the groups.

We now have to relate the coefficients and probabilities occurring in the transfer equations to operationally meaningful quantities and derive relations expressing these quantities. The coefficients and probabilities will be given for a two-spacial dimensional model. The preceding model and theory have been presented in three-dimensional form. There are two virtues in this approach. The first is an emphasis on generality and the second is to distinguish clearly the nature of physical entities in similar applications and the operational entities that will be presented below. For instance, there is no conceptual difficulty in defining our two-dimensional density. The two-dimensional equations to be considered are similar to those already derived except for the occurrence of a factor of 2π in place of 4π (see equation 1.2.9) with the obvious changes in dimensions.

GENERAL SIGNIFICANCE OF THE DENSITY ρ AND MASS ABSORPTION COEFFICIENT k

Strictly, the density $\rho \equiv \lim_{A \to O} \frac{M}{A}$

where A is an area and M is the mass associated with this area. As we are interested in a stochastic model, average values and probabilities apply, so define

$$\rho \equiv \frac{\overline{M}}{\overline{A}} \equiv \lim_{A \to O} \frac{M}{A}$$

where $\overline{M}$ is the total mass associated with a fixed area $\overline{A}$.

Recall that $I_V(\vec{R}, \vec{\Omega})\, d\sigma\, d\omega\, dt$ = units at $\vec{R}$ traveling in direction $\vec{\Omega}$.

and $k I_V(\vec{R}, \vec{\Omega})\, d\omega\, dt\, dM$ = loss of units at $\vec{R}$ traveling in direction $\vec{\Omega}$.

Thus, take $k \rho\, ds$ to be dimensionless.

Now $\rho \equiv$ mass/unit area, where mass is defined in terms of intensity of enemy action, terrain properties, etc.

k is thus seen to be a probability that in a unit area an absorption will take place

per unit mass

per unit length.

k can be called the mass absorption coefficient.

OPERATIONAL DEFINITION OF THE DENSITIES AND MASS ABSORPTION COEFFICIENT

We now consider separately the different absorption terms given in the table (page 10).

Annihilation by Enemy Action

Define

$\rho_{av1} \equiv$ occurrences unit area, where an occurrence could be the detonation of a bomb. As discussed,

2.3.1 $$\rho_{av1} \equiv \frac{\text{total number of expected events (occurrences) in a region}}{\text{area of the region}}.$$

Then k_{av1} is the probability per unit occurrence that a unit will be annihilated in terms of length $\perp$ to $\vec{\Omega}$.

ρ_{av2} and k_{av2} are similarly defined.

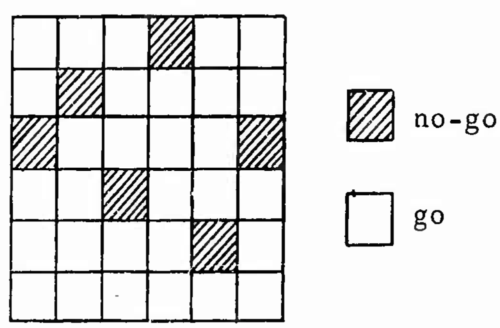
Speed Transfer by Enemy Action

k_{bv1} and k_{bv2} are just the probabilities per unit occurrence that a speed transfer will take place.

Speed Transfer by Terrain

The nature of terrain is fundamentally different in the cases of on-road and off-road.

Off-Road: From terrain features such as slope and soil conditions, a go/no-go function can be evaluated for an area which will determine whether a specific vehicle type will or will not stall in this area. If we consider a region R (see Figure 7) and impose a grid on it giving go/no-go values for different cells,



R

Figure 7.

a density ρ_{bv_i} can be defined as

$$2.3.2 \quad \rho_{bvi} \equiv \frac{\text{number of no-go cells in the region}}{\text{area of the region}} .$$

Then k_{cvi} becomes the probability per unit no-go that a unit will be transferred given in terms of length $\perp$ to $\vec{\Omega}$.

<u>On-Road</u>: Consider a road net in a given region. Impose upon it a rectangular grid (see Figure 8) such that the map in each cell is connected and such that each crossroad falls within a cell. Now, in each cell of this grid, construct a linear approximation to the road map such that the maximum distance from the road to the linear segment is a preassigned value, say ϵ (see Figure 9), and such that the segment is tangent

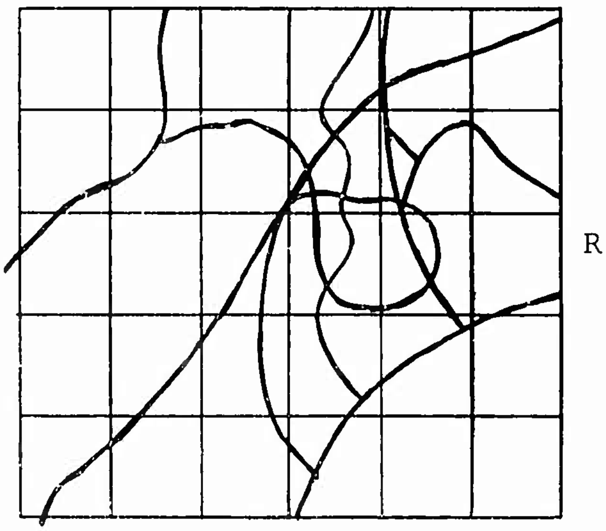


Figure 8.

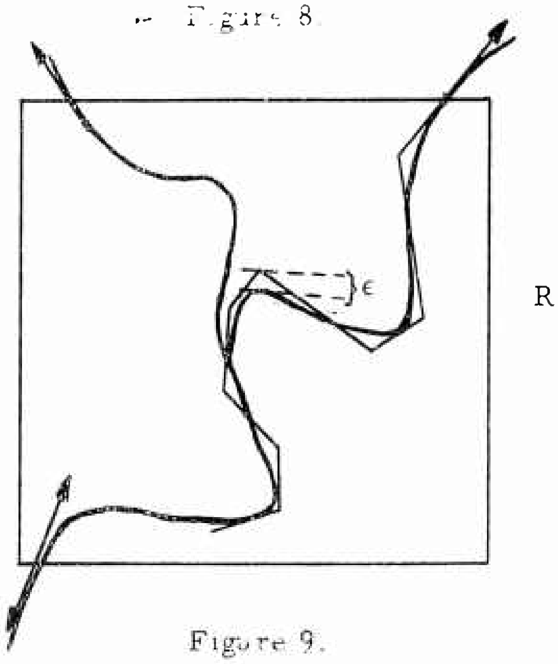


Figure 9.

to the road at its contact points. Now consider the following two quantities:

Φ = sum of the angles that adjacent segments made with each other then considered as vectors located in first and second quadrant with respect to a preferred direction y.

Ψ = the absolute value of the signed sum of the angles that the segments make with a preferred direction when located as above.

Further consider the conditions of the road net in a cell. Because of such factors as a bridge out, road demolished, or mud, traffic may be stalled. From these three quantities can be determined estimates of a transfer/no-transfer function for each cell. Thus, as in the case of off-road traffic, a density ρ_{bv2} and a mass absorption coefficient k_{cv2} can be evaluated.

Scattering by Enemy Action

k_{dv1} and k_{dv2} are just the probabilities per unit occurrence that a scattering event will take place.

Scattering by Terrain

k_{ev1} is the probability per unit no-go that a scattering event will take place. The on-road density ρ_{ev2} is, however, in general different from ρ_{bv2}. Consider Figure 9; a scattering event occurs if there are two or more branches issuing from a cell with different tangent vectors. We will assume that the case in which two road branches issue from a cell is fundamentally the same as that in which multibranches issue from a cell. A further refinement would be to make this distinction. Thus we could, in principle, determine a scatter/no-scatter function for each cell and then derive a density ρ_{ev2} and corresponding coefficient k_{ev2}.

GENERAL SIGNIFICANCE OF THE PHASE FUNCTIONS

The phase function $\frac{p(\vec{\Omega}, \vec{\Omega}')}{2\pi}$ was defined to be the probability per unit solid angle about $\vec{\Omega}'$ at $\vec{R}$ that a unit absorbed at $\vec{R}$ will be scattered into the $\vec{\Omega}$ direction (see equations 1.2.9 and 1.2.10).

OPERATIONAL DEFINITIONS OF THE PHASE FUNCTIONS

We now consider separately the different phase functions given in the table.

(β) Speed Transfer by Enemy Action

Here the on-road transfer phase function differs from the off-road transfer phase function. Realistic plan functions can be constructed in the following manner:

First recall that

$\frac{p(\vec{\Omega}, \vec{\Omega}')}{2\pi} d\omega$ = the probability per unit solid angle about $\vec{\Omega}'$ that the unit which was absorbed will be emitted into $d\omega$ about $\vec{\Omega}$, given that an absorption-scattering (transfer) event has occurred.

Now, consider $p_a(v_2 \to v_1, \vec{\Omega}, \vec{\Omega}')$ and consider the situation illustrated in Figure 10.

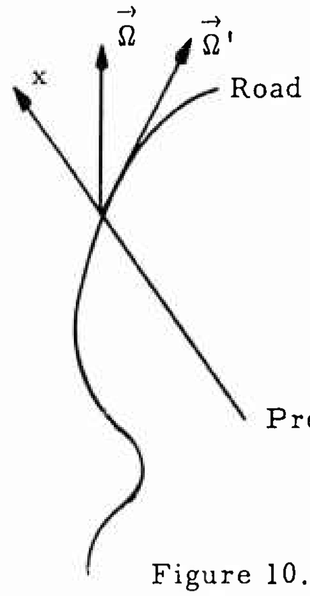


Figure 10.

Once a transferring event has occurred on the road, a unit traveling in the $\vec{\Omega}'$ direction becomes reoriented in the $\vec{\Omega}$ direction, with a symmetric distribution about the preferred direction of travel, x. As with the phase function for pure scattering to be considered later, we assume that $p_a(v_2 \to v_1, \vec{\Omega}, \vec{\Omega}') = p_a(v_2 \to v_1, \vec{\Omega})$, or the phase function is independent of $\vec{\Omega}'$. Thus,

2.5.1
$$\boxed{\begin{aligned} &\frac{1}{2\pi} \int_{\omega'} p_a(v_2 \to v_1, \vec{\Omega})\, I_{v2}(\vec{R}, \vec{\Omega}')\, d\omega' \\ &= \frac{p_a(v_2 \to v_1, \vec{\Omega})}{2\pi} \int_{\omega'} I_{v2}(\vec{R}, \vec{\Omega}')\, d\omega' \end{aligned}} .$$

Now consider $p_a(v_1 \rightarrow v_2, \vec{\Omega}, \vec{\Omega}')$ and consider the situation illustrated in Figure 11.

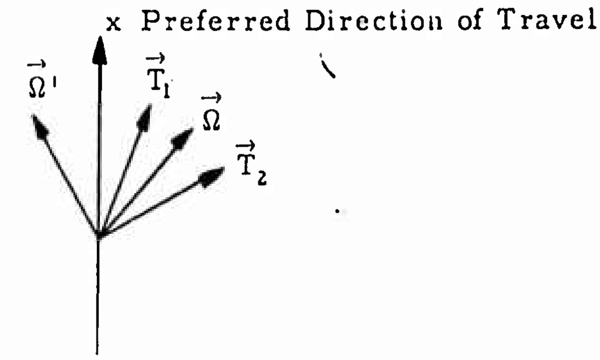


Figure 11.

Once a transferring event has occurred, a unit traveling in the $\vec{\Omega}'$ direction becomes reoriented in the $\vec{\Omega}$ direction independent of $\vec{\Omega}'$. Let the T_i's be the tangent vectors to the roads as explained in paragraph e under "Speed Transfer by Terrain", where the road has been "linearized", and let $\vec{\Omega}$ be the resultant on-going direction of the units. We assume that the phase function is of the following form:

$$p_a(v_1 \rightarrow v_2, \vec{\Omega}, \vec{\Omega}') = \sum_{i=1}^{\mu} \bar{\delta}_a(\vec{T}_i(\vec{R}), \vec{\Omega})$$

where

$$\bar{\delta}_a(\vec{T}_i(\vec{R}), \vec{\Omega}) = \begin{cases} 0 \text{ if } \dfrac{\vec{T}_i(\vec{R})}{\|\vec{T}_i\|} \neq \vec{\Omega} \\ a_i \text{ if } \dfrac{\vec{T}_i}{\|\vec{T}_i\|} = \vec{\Omega}, \end{cases}$$

μ is the number of road branches, and

$$\int_\omega \sum_{i=1}^{\mu} \bar{\delta}_a(\vec{T}_i, \vec{\Omega})\, d\omega = 1.$$

A realistic determination of the $\bar{\delta}_a(\vec{T}_i, \vec{\Omega})$ is indicated in Figure 12.

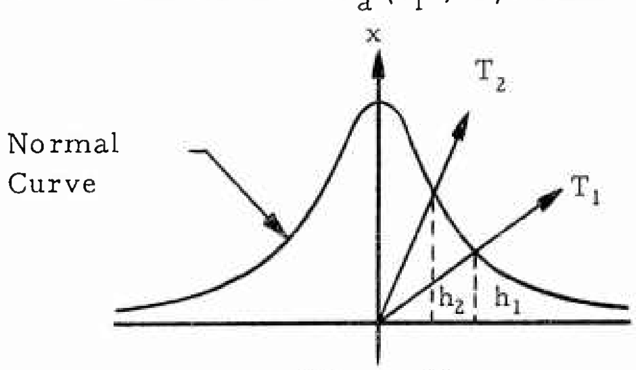


Figure 12.

when $\bar{\delta}_a(\vec{T}_1, \vec{\Omega})$ is to $\bar{\delta}_a(\vec{T}_2, \vec{\Omega})$ as h_1 is to h_2

and where $\int_\omega \left\{ \bar{\delta}_a(\vec{T}_1, \vec{\Omega}) + \delta_a(\vec{T}_2, \vec{\Omega}) \right\} d\omega = 1.$

Thus,

$$\frac{1}{2\pi} \int_{\omega'} p_a(v_1 \rightarrow v_2, \vec{\Omega}, \vec{\Omega}') \, I_{v1}(\vec{R}, \vec{\Omega}') \, d\omega' = \frac{\sum_{i=1}^{\mu} \bar{\delta}_a(\vec{T}_1, \vec{\Omega})}{2\pi}$$

2.5.2 $$\int_{\omega'} I_{v1}(\vec{R}, \vec{\Omega}') \, d\omega' .$$

γ Speed Transfer by Terrain

As with speed transfer in part (β) above, we obtain:

2.5.3 $$\frac{1}{2\pi} \int_{\omega'} p_b(v_2 \rightarrow v_1, \vec{\Omega}, \vec{\Omega}') \, I_{v1}(\vec{R}, \vec{\Omega}') \, d\omega' = \frac{p_b(v_2 \rightarrow v_1, \vec{\Omega})}{2\pi} \int_{\omega'} I_{v2}(\vec{R}, \vec{\Omega}') \, d\omega'$$

and

2.5.4 $$\frac{1}{2\pi} \int_{\omega'} p_b(v_1 \rightarrow v_2, \vec{\Omega}, \vec{\Omega}') \, I_{v1}(\vec{R}, \vec{\Omega}') \, d\omega' = \frac{\sum_{i=1}^{\mu} \bar{\delta}_b(\vec{T}_i, \vec{\Omega})}{2\pi} \int_{\omega'} I_{v1}(\vec{R}, \vec{\Omega}') \, d\omega'$$

where the $\bar{\delta}_b$'s, refer to $p_b(v_1 \rightarrow v_2)$.

(δ) Scattering by Enemy Action

Here the on-road case differs from the off-road case.

Off Road: We have determined, considering that a unit is goal-oriented, that a realistic phase function can be determined in the following manner: To fix discussion, let the positive x direction in a rectangular system be the "ideal" direction of traffic (see Figure 13).

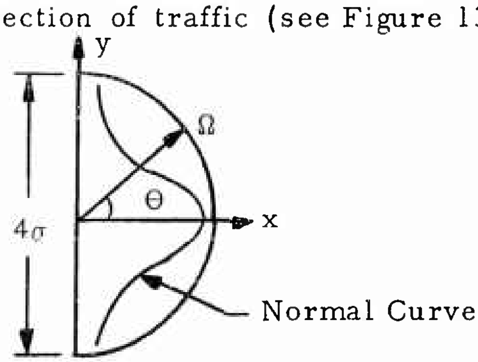


Figure 13.

Assume that once a scattering "event" has taken place, the unit independent of $\vec{\Omega}'$ seeks to orient itself in the positive x direction resulting in an $\vec{\Omega}$ orientation.

Thus,

2.5.5. $$\frac{p(\vec{\Omega}, \vec{\Omega}')}{2\pi} = \frac{p(\vec{\Omega})}{2\pi} \quad \text{and}$$

therefore we obtain

2.5.6 $$\boxed{\frac{1}{2\pi} \int_{\omega'} p_{cv1} (\vec{\Omega})\, I_{v1} (\vec{R}, \vec{\Omega}')\, d\omega' = \frac{p_{cv1} (\vec{\Omega})}{2\pi} \int_{\omega'} I_{v1} (\vec{R}, \vec{\Omega}')\, d\omega'} \, .$$

<u>On Road</u>: To a first approximation, on-road scattering by enemy action does not take place.

Therefore,

2.5.7 $$\frac{1}{2\pi} \int_{\omega'} p_{cv2} (\vec{\Omega}, \vec{\Omega}')\, I_{v2} (\vec{R}, \vec{\Omega}')\, d\omega' \equiv 0.$$

<u>(ϵ) Scattering by Terrain</u>

Here the on-road case differs from the off-road case.

<u>Off Road</u>: We assume that, fundamentally, the phase function for terrain scattering is of the same form as that for enemy action.

Thus,

2.5.8 $$\frac{1}{2\pi} \int_{\omega'} p_{dv1} (\vec{\Omega})\, I_{v1} (\vec{R}, \vec{\Omega}')\, d\omega' = \frac{p_{dv1} (\vec{\Omega})}{2\pi} \int_{\omega'} I_{v1} (\vec{R}, \vec{\Omega}')\, d\omega'.$$

<u>On Road</u>: The on-road scattering phase function might be viewed as in the cases of speed transfer due to enemy action and due to terrain. Consequently, we obtain

2.5.9 $$\frac{1}{2\pi} \int_{\omega'} p_{dv2} (\vec{\Omega}, \vec{\Omega}')\, I_{v2} (\vec{R}, \vec{\Omega}')\, d\omega' =$$

$$\frac{\sum_{i=1}^{\mu} \bar{\delta}_d (\vec{T}_i, \vec{\Omega})}{2\pi} \int_{\omega'} I_{v2} (R, \Omega')\, d\omega' .$$

THE MODIFIED GENERAL EQUATIONS

Clearly, the definitions and assumptions introduced in this chapter mathematically simplify the general transfer equations. The appropriate simplified equations will be derived below.

Time-Independent Equations

The absorption terms given in the section "Notes on the Specific Transport Equation" in Chapter 1 are unaltered. The emission terms will take on the following form:

$$j_{\alpha v1} \; (v_2 \rightarrow v_1) = \frac{k_{bv2}}{2\pi} p_a \; (v_2 \rightarrow v_1, \vec{\Omega}) \int_{\omega'} I_{v2}(\vec{R}, \vec{\Omega}') \, d\omega'$$

$$j_{\alpha v2} \; (v_1 \rightarrow v_2) = \frac{k_{bv1}}{2\pi} \sum_{i=1}^{\mu} \bar{\delta}_a \; (\vec{T}_i, \vec{\Omega}) \int_{\omega'} I_{v1} \; (\vec{R}, \vec{\Omega}') \, d\omega'$$

$$j_{\beta v1} \; (v_2 \rightarrow v_1) = \frac{k_{cv2}}{2\pi} \; p_b \; (v_2 \rightarrow v_1, \vec{\Omega}) \int_{\omega'} I_{v2} \; (\vec{R}, \vec{\Omega}') \, d\omega'$$

$$j_{\beta v2} \; (v_1 \rightarrow v_2) = \frac{k_{cv1}}{2\pi} \sum_{i=1}^{\mu} \bar{\delta}_b \; (\vec{T}_i, \vec{\Omega}) \int_{\omega'} I_{v1} \; (\vec{R}, \vec{\Omega}') \, d\omega'$$

$$j_{\gamma v1} = \frac{k_{dv1}}{2\pi} \; p_{cv1} \; (\vec{\Omega}) \int_{\omega'} I_{v1} \; (\vec{R}, \vec{\Omega}') \, d\omega'$$

$$j_{\gamma v2} \equiv 0$$

$$j_{\delta v1} = \frac{k_{ev1}}{2\pi} p_{dv1} \; (\vec{\Omega}) \int_{\omega'} I_{v1} \; (\vec{R}, \vec{\Omega}') \, d\omega'$$

$$j_{\delta v2} = \frac{k_{ev2}}{2\pi} \sum_{i=1}^{\mu} \bar{\delta}_d \; (\vec{T}_i, \vec{\Omega}) \int_{\omega'} I_{v2} \; (\vec{R}, \vec{\Omega}') \, d\omega'.$$

Thus,

2.6.1

$$\frac{dI_{v1}(\vec{R},\vec{\Omega})}{ds} = -K_{v1}\, I_{v1}\,(\vec{R},\vec{\Omega}) + \Sigma_{tv1} \int_{\omega'} I_{v2}\,(\vec{R},\vec{\Omega}')\, d\omega' + \Sigma_{sv1} \int_{\omega'} I_{v1}\,(\vec{R},\vec{\Omega}')\, d\omega'$$

AND

$$\frac{dI_{v2}(\vec{R},\vec{\Omega})}{ds} = -K_{v2}\, I_{v2}\,(\vec{R},\vec{\Omega}) + \Sigma_{tv2} \int_{\omega'} I_{v1}\,(\vec{R},\vec{\Omega})\, d\omega' + \Sigma_{sv2} \int_{\omega'} I_{v2}\,(\vec{R},\vec{\Omega})\, d\omega'$$

where

$$\Sigma_{tv1} = \frac{k_{bv2}\,\rho_{av2}}{2\pi}\, p_a\,(v_2 \to v_1\, \vec{\Omega}\,) + \frac{k_{cv2}\,\rho_{bv2}}{2\pi}\, p_b\,(v_2 \to v_1\,, \vec{\Omega}\,)$$

$$\Sigma_{sv1} = \frac{k_{dv1}\,\rho_{av1}}{2\pi}\, p_{cv1}\,(\vec{\Omega}) + \frac{k_{ev1}\,\rho_{bv1}}{2\pi}\, p_{dv1}\,(\vec{\Omega})$$

$$\Sigma_{tv2} = \frac{k_{bv1}\,\rho_{av1}}{2\pi} \sum_{i=1}^{\mu} \bar{\delta}_a\,(\vec{T}_i\,, \vec{\Omega}) + \frac{k_{cv1}\,\rho_{bv1}}{2\pi} \sum_{i=1}^{\mu} \bar{\delta}_b\,(\vec{T}_i\,, \vec{\Omega})$$

$$\Sigma_{sv2} = \frac{k_{ev2}\,\rho_{ev2}}{2\pi} \sum_{i=1}^{\mu} \bar{\delta}_d\,(\vec{T}_i\,, \vec{\Omega})\,.$$

Time-Dependent Equations

The time-dependent equations become:

2.6.2

$$\frac{1}{v_1}\frac{dI_{v1}(\vec{R},\vec{\Omega})}{dt} = -\frac{dI_{v1}(\vec{R},\vec{\Omega})}{ds} - K_{v1}\, I_{v1}\,(\vec{R},\vec{\Omega}) + \Sigma_{tv1} \int_{\omega'} I_{v2}\,(\vec{R},\vec{\Omega}')\, d\omega' + \Sigma_{sv1} \int_{\omega'} I_{v1}\,(\vec{R},\vec{\Omega}')\, d\omega'$$

$$\frac{1}{v_2}\frac{dI_{v2}(\vec{R},\vec{\Omega})}{dt} = -\frac{dI_{v2}(\vec{R},\vec{\Omega})}{ds} - K_{v2}\, I_{v2}\,(\vec{R},\vec{\Omega}) + \Sigma_{tv2} \int_{\omega'} I_{v1}\,(\vec{R},\vec{\Omega})\, d\omega' + \Sigma_{sv2} \int_{\omega'} I_{v2}\,(\vec{R},\vec{\Omega}')\, d\omega'\,.$$

In general, we will be interested in the density functions that are consequently governed by the following equations:

$$\boxed{\begin{aligned} \frac{1}{v_1}\frac{du_1}{dt} &= -\frac{du_1}{ds} - K_1 u_1 + \bar{\Sigma}_{t1} u_2 + \bar{\Sigma}_{s1} u_1 \\ \frac{1}{v_2}\frac{du_2}{dt} &= -\frac{du_2}{ds} - K_2 u_2 + \bar{\Sigma}_{t2} u_1 + \bar{\Sigma}_{s2} u_2 \end{aligned}} \tag{2.6.3}$$

where

$$u_1 = u_1 (\vec{R}) \qquad u_2 = u_2 (\vec{R})$$

$$\bar{\Sigma}_{t1} = \frac{k_{bv2}\ \rho_{av2}}{2\pi} + \frac{k_{cv2}\ \rho_{bv2}}{2\pi}$$

$$\bar{\Sigma}_{s1} = \frac{k_{dv1}\ \rho_{av1}}{2\pi} + \frac{k_{ev1}\ \rho_{bv1}}{2\pi}$$

$$\bar{\Sigma}_{t2} = \frac{k_{bv1}\ \rho_{av1}}{2\pi} + \frac{k_{cv1}\ \rho_{bv1}}{2\pi}$$

$$\bar{\Sigma}_{s2} = \frac{k_{ev2}\ \rho_{ev2}}{2\pi}$$

where arbitrary source and loss terms have been dropped.

BIBLIOGRAPHY

1. Ambarzumian, V. A., *Theoretical Astrophysics*, Pergamon Press, New York, 1958.

2. Anselone, Philip, A., "The Uniform Convergence of Chandrasekhar's Approximations in Classical Transfer Problems", Mathematical Research Center Summary Report 97, University of Wisconsin, Madison, Wisconsin, October 1959.

3. Chandrasekhar, S., *Radiative Transfer*, Dover Publications, New Yord, N. Y., 1960.

4. Glasstone, S., and Edlund, M. C., *The Elements of Nuclear Reactor Theory*, D. Van Nostrand Company, Inc., New York, N. Y., 1952.

5. Morse, P. M., and Feshbach, Herman, *Methods of Theoretical Physics*, McGraw-Hill Book Company, New York, N. Y., 1953.

6. "Notes Paralleling S. Chandrasekhar", attributed to P. Anselone.

7. Weinberg, A. M., and Wigner, E. P., *The Physical Theory of Neutron Chain Reactors*, University of Chicago Press, Chicago, Illinois, 1958.

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